✓

Exponents and Scientific Notation

1. Use the example of the first row of the chart below, Multiplication of Like Bases, to fill in the rest of the chart. This chart assumes that a and b are real numbers, and $a \neq 0$. m and n represent integers.

Properties of Exponents

Description	Example	Expanded Form	Property
Multiplication of Like Bases	$a^3 \cdot a^5$	$a^3 \cdot a^5 = (a \cdot a \cdot a)(a \cdot a \cdot a \cdot a \cdot a) = a^{3+5} = a^7$	$a^n \cdot a^m = a^{n+m}$
Division of Like Bases	$\frac{a^6}{a^2}$	$\frac{a^6}{a^2} = a^4$	$\frac{a^n}{a^m} = a^{(n-m)}$
Power Rule	$(a^2)^3$	$(a^2)^3 = a^6$	$(a^n)^m = a^{nm}$
Power of a Product	$(ab)^3$	$(ab)^3 = a^3b^3$	$(ab)^n = a^nb^n$
Power of a Quotient	$\left(\frac{b}{a}\right)^3$	$\left(\frac{b}{a}\right)^3 = \frac{b^3}{a^3}$	$\left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

Definition: For any real number a, $a \neq 0$, $a^0 = 1$. We are going to use this definition to come up with a rule for negative exponents.

$a^{-n} = a^{0-n} = \frac{a^0}{a^n} = \frac{1}{a^n}$ so $a^{-n} = \frac{1}{a^n}$.

2. Use the rules and definitions above to simplify the following, leaving your answers with positive exponents only.

5^{-2} $= \frac{1}{25}$

$\left(\frac{2}{3}\right)^{-1}$ $= \frac{3}{2}$

$(-2)^{-3}$ $= -\frac{1}{8}$

-2^{-3} $= -\frac{1}{8}$

$(10ab)^0$ $= 1$

$10ab^0$ $= 10a$

$y^3 \cdot y^7$ $= y^{10}$

$\frac{x^{11}}{x^4}$ $= x^7$

$(3z^2)^4$ $= 81z^8$

7^2q^{-3} $\frac{49}{q^3}$

$\dfrac{r}{r^{-1}}$

$= r^2$

$\dfrac{p^2q}{p^5q^{-1}}$

$= \dfrac{q^2}{p^3}$

$\dfrac{25x^2y^{12}}{10x^5y^7}$

$= \dfrac{5y^5}{2x^3}$

$(-6a^{-2}b^3c)^{-2}$

$= \dfrac{1}{(-6a^{-2}b^3c)^2}$

$= \dfrac{a^4}{36b^6c^2}$

$(mn^3)^2(5m^{-2}n^2)$

$= \cancel{m^2}n^6 \cdot \dfrac{5n^2}{\cancel{m^2}}$

$= 5n^8$

$\left(\dfrac{a}{b^2}\right)^2(3a^2b^3)$

$= \dfrac{a^2}{b^4} \cdot 3a^2b^3$

$= \dfrac{3a^4}{b}$

$3xy^5\left(\dfrac{2x^4y}{6x^5y^3}\right)^{-2}$

$= 3xy^5 \cdot \dfrac{36x^{10}y^6}{4x^8y^2}$

$= 3xy^5 \cdot 9x^2y^4$

$= 27x^3y^9$

A number expressed in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer, is said to be in **scientific notation.**

3. Write each of the following in "proper" scientific notation.

103

1.03×10^2

0.00037

3.7×10^{-4}

0.0435×10^{-5}

$4.35 \times 10^{-2} \times 10^{-5}$

$= 4.35 \times 10^{-7}$

682×10^4

$6.82 \times 10^2 \times 10^4$

$= 6.82 \times 10^6$

4. Perform the indicated operation and leave your answer in scientific notation.

$(6.5 \times 10^3)(5.2 \times 10^{-8})$

33.8×10^{-5}

$= 3.38 \times 10 \times 10^{-5}$

$= 3.38 \times 10^{-4}$

$\dfrac{3 \times 10^{13}}{1.5 \times 10^5}$

$= 2 \times 10^8$

$\dfrac{1.32 \times 10^{-2}}{1.2 \times 10^{-15}}$

$= 1.1 \times 10^{15} \times 10^{-2}$

$= 1.1 \times 10^{13}$

✓

Polynomial Addition, Subtraction, and Multiplication

Definitions: A **polynomial** in x is defined as a finite sum of terms of the form ax^n, where a is a real number and is called the **coefficient** of the term, and n is a whole number and is the **degree of the term**.

1. State the coefficient and degree of the term.

$5x^4$	$\frac{2}{7}x$	x^{11}	21
C: 5	C = $\frac{2}{7}$	C = 1	C = 21
d: 4	d: 1	d = 11	d = 0

Definitions: If a polynomial has exactly one term, it is called a **monomial**; a two terms polynomial is a **binomial**; and a three term polynomial is a **trinomial**. Typically, we write a polynomial in descending order, starting with the term of largest degree, called the **leading term**. Its coefficient is called the **leading coefficient**. The **degree of the polynomial** is the degree of its highest term, the leading term.

2. Write the given polynomial in descending order, state the leading coefficient and the degree of the polynomial.

$w + 5 - 4w^3 + 7w^5$	$13y - y^2$	$2.5a^5 - a^9 + 2a^4$
$7w^5 - 5w^3 + w + 5$	$-y^2 + 13y$	$-a^9 + 2.5a^5 + 2a^4$
d: 5 C: 7	d: 2 C: -1	d: 9 C: -1

Polynomials may have more than one variable, and in such a case, the degree of a term is the sum of the exponents of the variables contained in the term.

3. What is the degree of the following polynomial? $2x^2y^2z^5 - 3xy^5z^5 + 12xyz^{10}$

d: 12

4. When adding or subtracting polynomials, you combine **like terms**. Simplify the following expressions.

$(11ab - 23b^2) + (7ab - 19b^2)$

$= 18ab - 42b^2$

$(8y^2 - 4y^3) - (3y^2 - 8y^3)$

$= 8y^2 - 4y^3 - 3y^2 + 8y^3$

$= 4y^3 + 5y^2$

$(-8x^3 + 6x + 7) - (-4 - 5x^3)$

$= -8x^3 + 6x + 7 + 4 + 5x^3$

$= -3x^3 + 6x + 11$

$(-2x^2y^2 + 6xy^2 + 7xy) - (5xy^2 - 2xy - 4)$

$= -2x^2y^2 + 6xy^2 + 7xy - 5xy^2 + 2xy + 4$

$= -2x^2y^2 + xy^2 + 9xy + 4$

$(-ab + 5a^2b) + [7ab^2 - 2ab - (7a^2b + 2ab^2)]$

$= -ab + 5a^2b + 7ab^2 - 2ab - 7a^2b - 2ab^2$

$= -2a^2b + 5ab^2 - 3ab$

Now on to multiplication. We have already seen our rules of exponents, which we have used to simplify expressions like $(2x^3y^4)(3xy^2) = 6x^4y^6$. We will use that concept, along with the distribution property and the addition/subtraction simplifying we practiced just now to multiply and simplify polynomials.

5. Multiply and simplify the following expressions.

$2m^3n^2(m^2n^3 - 3mn^2 + 4n)$

$= 2m^5n^5 - 6m^4n^4 + 8m^3n^3$

$3xy - 4x(2x^2y - 5y + 3x^2y^2)$

$= 3xy - 8x^3y + 20xy - 12x^3y^2$

$= 23xy - 8x^3y - 12x^3y^2$

$(x-3)(x+4)$

$= x^2 + 4x - 3x - 12$

$= x^2 + x - 12$

$(2x+3y)(5x-y)$

$= 10x^2 - 2xy + 15xy - 3y^2$

$= 10x^2 + 13xy - 3y^2$

$(w+4)(w-4)$

$= w^2 - 4w + 4w - 16$

$= w^2 - 16$

$(x+7)^2$

$= x^2 - 14x + 49$

$(2x+y)(x^2 - 4xy + 6y^2)$

$= 2x^3 - 8x^2y + 12xy^2$
$+ x^2y - 4xy^2 + 6y^3$

$= 2x^3 - 7x^2y + 8xy^2 + 6y^3$

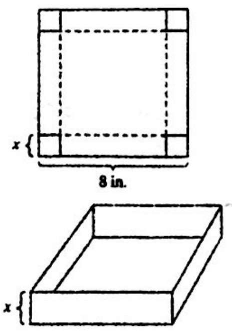
$(x+3)(x^2 - 3x + 9)$

$= x^3 - 3x^2 + 9x + 3x^2 - 9x + 27$

$= x^3 + 27$

6. A box is created from a square piece of cardboard with sides that are 8 inches in length. The box is created by cutting a square from each corner and folding up the sides (see the diagram below). Let x represent the length of the sides of the squares removed from each corner.

   a. Write a function representing the volume of the box.
   b. Find the volume if 1 inch squares are removed from the corners.



a: $v = (8-2x)^2 \cdot x$

$= (64 - 32x + 4x^2) \cdot x = \boxed{64x - 32x^2 + 4x^3}$

b: v : 36 in

Since, $x = 1 \Rightarrow v = (8 - 2(1))^2 \cdot (1)$

$= \boxed{36 \text{ in}}$

✓

Polynomial Division

1. Division by a monomial. Divide the polynomials, and check your answer with multiplication.

$(36y + 24y^2 + 6y^3) \div (3y)$

$6y^3 + 24y^2 + 36y \,\big|\, 3y$

$2y^2 + 8y + 12$

$-6y^3$

$24y^2 + 36y$

$-24y^2$

$36y$

$-36y$

0

check:

$(2y^2 + 8y + 12)(3y) = 6y^3 + 24y^2 + 36y$ ✓

$(25m^5n - 10m^4n + m^3) \div (5m^3n)$

$\frac{25m^5n}{5m^3n} - \frac{10m^4n}{5m^3n} + \frac{m^3}{5m^3n} = \boxed{5m^2 - 2m + \frac{1}{5n}}$

check: $(5m^2 - 2m + \frac{1}{5n})(5m^3n) =$

$= 25m^5n - 10m^4n + m^3$ ✓

2. Division by a binomial. Divide the polynomials by using long division. Check your answer by multiplication. Be sure your polynomial is written in descending order, and it is easiest to use place holders like $0x^n$ for any missing terms.

$(3x^3 + 2x^2 - 7x + 2) \div (x + 2)$

$3x^3 + 2x^2 - 7x + 2 \,\big|\, x + 2$

$3x^2 - 4x + 1$

$-3x^3 - 6x^2$

$-4x^2 - 7x + 2$

$4x^2 + 8x$

$x + 2$

$-x - 2$

0

check:

$(3x^2 - 4x + 1)(x + 2) = 3x^3 \underbrace{- 4x^2} + x \underbrace{+ 6x^2} - 8x + 2$

$= 3x^3 + 2x^2 - 7x + 2$ ✓

$(-6x + 8x^3 + 22) \div (2x - 1)$

$8x^3 - 6x + 22 \,\big|\, 2x - 1$

$4x^2 + 2x - 2$

$-8x^3 + 4x^2$

$4x^2 - 6x + 22$

$-4x^2 + 2x$

$-4x + 22$

$+4x - 2$

20

check:

$(4x^2 + 2x - 2)(2x - 1) + 20 =$

$= 8x^3 + 4x^2 - 4x - 4x^2 - 2x + 2 + 20$

$= 8x^3 - 6x + 22$ ✓

3. **Divide the following. Check your answers with multiplication.**

$(81x^4 - 1) \div (3x + 1)$

$81x^4 - 1 \;|\; 3x+1$

$-81x^4 - 27x^3 \quad 27x^3 - 9x^2 + 3x - 1$

$-27x^3 - 1$

$+27x^3 + 9x^2$

$9x^2 - 1$

$-9x^2 - 3x$

$-3x - 1$

$+3x + 1$

0

check:

$(27x^3 - 9x^2 + 3x - 1)(3x + 1)$

$= 81x^4 - 27x^3 + \cancel{9x^2} - \cancel{3x} + 27x^3 - \cancel{9x^2} + \cancel{3x} - 1 = 81x^4 - 1$ ✓

$(x^4 - x^3 - x^2 + 4x - 2) \div (x^2 + x - 1)$

$x^4 - x^3 - x^2 + 4x - 2 \;|\; x^2 + x - 1$

$-x^4 - x^3 + x^2 \quad x^2 - 2x + 2$

$-2x^3 + 4x - 2$

$+2x^3 + 2x^2 - 2x$

$2x^2 + 2x - 2$

$-2x^2 - 2x + 2$

0

check:

$(x^2 - 2x + 2)(x^2 + x - 1)$

$= x^4 - 2x^3 + \cancel{2x^2} + x^3 - \cancel{2x^2} + 2x - x^2 + 2x - 2 = x^4 - x^3 - x^2 + 4x - 2$ ✓

$(2m^3 - 4m^2 + 5m - 33) \div (m - 3)$

$2m^3 - 4m^2 + 5m - 33 \;|\; m - 3$

$-2m^3 + 6m^2 \quad 2m^2 + 2m + 11$

$2m^2 + 5m - 33$

$-2m^2 + 6m$

$11m - 33$

$-11m + 33$

0

check:

$(2m^2 + 2m + 11)(m - 3)$

$= 2m^3 + 2m^2 + 11m - 6m^2 - 6m - 33$

$= 2m^3 - 4m^2 + 5m - 33$ ✓